Effects of Ions on the Propagation of Langmuir Oscillations in Cold Quantum Electron-Ion Plasmas

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The effects of ions on the propagation of Langmuir oscillations are investigated in cold quantum electron-ion plasmas. It is shown that the higher and lower frequency modes of the Langmuir oscillations would propagate in cold quantum plasmas according to the effects of ions. It is also shown that these two propagation modes merge into one single propagation mode if the contribution of ions is neglected. It is found that the quantum effect enhances the phase and group velocities of the higher frequency mode of the propagation. In addition, it is shown that the phase velocity of the lower frequency mode is saturated with increasing the quantum wavelength and further that the group velocity of the lower frequency mode has a maximum position in the domains of the wave number and quantum wavelength.

Key words: Effects of Ions; Langmuir Oscillations; Cold Quantum Plasmas.

There has been considerable interest in investigating various properties of quantum plasmas [1 – 4] since the quantum plasmas have been used in the development of modern sciences and technologies such as nanowires, laser-produced plasmas, quantum dots, quantum wells, and semiconductor devices. It has been well known that the Langmuir oscillation in bulk cold classical plasmas can not be propagated and shows only electrostatic oscillation modes. Very recently, it was found that the Langmuir oscillations in quantum plasmas can be propagated due to the quantum effects caused by the multiparticle quantum Bohm potential [5]. In cold classical plasmas, we can readily prove that the contribution of ions slightly enhances the frequency of the Langmuir oscillation. However, the effects of ions in cold quantum plasmas would be quite different from those in cold classical plasmas due to the correlation of quantum effects and the contribution of ions. Thus, in this paper we investigate the effects of ions on the propagation of Langmuir oscillations in cold quantum electron-ion plasmas since the dispersion properties of Langmuir oscillations would provide a useful tool for investigating the physical properties of quantum plasmas.

It has been known that the plasma dielectric function (ε_c) of bulk cold classical electron (e)-ion (i) plasmas [6], i. e., neglecting the pressure gradient effects, is given by

$$\varepsilon_c(\omega) = 1 - \sum_{\alpha=\mathrm{e,i}} \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2}, \tag{1}$$

where $\omega_{\mathrm{p}\alpha}[= (4\pi n_\alpha q_\alpha^2/m_\alpha)^{1/2}]$ is the plasma frequency of the species α, n_α is the number density, q_α is the electric charge, and m_α is the mass. Thus, the dispersion relation ($\varepsilon_c = 0$) of bulk cold classical electron-ion plasmas shows the non-propagating electrostatic oscillating mode with the frequency $\omega = (\omega_{\mathrm{pe}}^2 + \omega_{\mathrm{pi}}^2)^{1/2}$, so that the effects of ions just increase the oscillation frequency. In a very recent investigation [5], the plasma dielectric function (ε_q) of cold quantum plasmas including the Bohm potential terms [5, 7] proportional to $\nabla[(\nabla^2 n_\alpha^{1/2})/n_\alpha^{1/2}]$ is given by

$$\varepsilon_q(k,\omega) = 1 - \sum_{\alpha=\mathrm{e,i}} \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2} \frac{1}{1 - k^4\lambda_{\mathrm{q}\alpha}^4\omega_{\mathrm{p}\alpha}^2/\omega^2}, \tag{2}$$

where $\lambda_{\mathrm{q}\alpha}$ $[= (\hbar/2m_\alpha\omega_{\mathrm{p}\alpha})^{1/2}]$ is the quantum wavelength of the species α. Hence, the plasma dielectric function of cold quantum electron-ion plasmas would be approximated as

$$\varepsilon_q(k,\omega) \approx 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega^2 - k^4\lambda_{\mathrm{qe}}^4\omega_{\mathrm{pe}}^2} - \frac{\omega_{\mathrm{pi}}^2}{\omega^2}, \tag{3}$$

since $\omega_{\rm pe} \gg \omega_{\rm pi}$. The dispersion relation ($\varepsilon_{\rm q} = 0$) of bulk cold quantum plasmas is then obtained by

$$\omega^4 - \left(k^4\lambda_{\rm qe}^4\omega_{\rm pe}^2 + \omega_{\rm pe}^2 + \omega_{\rm pi}^2\right)\omega^2 + k^4\lambda_{\rm qe}^4\omega_{\rm pe}^2\omega_{\rm pi}^2 = 0. \quad (4)$$

This dispersion relation is expected to be reliable for $\sqrt{k_{\rm B}T_{\rm Fe}/\hbar\omega_{\rm pe}} < k\lambda_{\rm qe}$, where $k_{\rm B}$ is the Boltzmann constant and $T_{\rm Fe}$ is the Fermi electron temperature, since the electrons are assumed to be cold. If we neglect the contribution of ions in (4), the dispersion relation is reduced to the single oscillating mode: $\omega^2 = \omega_{\rm pe}^2(1 + k^4\lambda_{\rm qe}^4)$ [5]. After some algebraic manipulations, the two possible modes of the propagation of Langmuir oscillations in bulk cold quantum plasmas including the contribution of ions are found to be

$$(\omega/\omega_{\rm pe})_+^2 = \frac{1}{2}\Big\{k^4\lambda_{\rm qe}^4 + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1 + \Big[\Big(k^4\lambda_{\rm qe}^4 + 2k^2\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) \cdot \Big(k^4\lambda_{\rm qe}^4 - 2k^2\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big]^{1/2}\Big\}, \quad (5)$$

$$(\omega/\omega_{\rm pe})_-^2 = \frac{1}{2}\Big\{k^4\lambda_{\rm qe}^4 + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1 - \Big[\Big(k^4\lambda_{\rm qe}^4 + 2k^2\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) \cdot \Big(k^4\lambda_{\rm qe}^4 - 2k^2\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big]^{1/2}\Big\}. \quad (6)$$

A detailed discussion on the low frequency ion oscillation in an ultracold quantum plasma can be found in an excellent paper by Shukla and Stenflo [8]. Since the frequency depends on the wave number, the Langmuir oscillation propagates inside the cold quantum plasma. It is interesting to note that, if we neglect the contribution of ions in (5) and (6), the lower frequency mode $(\omega/\omega_{\rm pe})_-$ vanishes so that the propagation of the higher frequency mode $(\omega/\omega_{\rm pe})_+$ would only be possible. Thus, it is found that the contribution of ions supports two possible propagation modes of the Langmuir oscillations in cold quantum electron-ion plasmas. In addition, it should be noted that, when the quantum effects are neglected, the lower frequency mode $(\omega/\omega_{\rm pe})_-$ also vanishes and then the higher frequency mode $(\omega/\omega_{\rm pe})_+$ turns out to be the well-known electrostatic oscillating mode $[1 + (\omega_{\rm pi}/\omega_{\rm pe})^2]^{1/2}$ of cold classical electron-ion plasmas. Then, the group velocities of the higher and lower frequency modes including the contribution of ions and the quantum effects are, respectively, obtained as

$$\frac{{\rm d}\bar\omega_+}{{\rm d}\bar k} = 2^{-3/2}\Big\{4\bar k^3\bar\lambda_{\rm qe}^4 + \Big[\big(2\bar k^3\bar\lambda_{\rm qe}^4 + 2\bar k\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe})\big)\Big(\bar k^4\bar\lambda_{\rm qe}^4 - 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) + \big(2\bar k^3\bar\lambda_{\rm qe}^4 - 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe})\big) \cdot \Big(\bar k^4\bar\lambda_{\rm qe}^4 + 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big] \Big/ \Big[\Big(\bar k^4\bar\lambda_{\rm qe}^4 - 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) \cdot \Big(\bar k^4\bar\lambda_{\rm qe}^4 + 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big]^{1/2}\Big\} \Big/ \Big\{\bar k^4\bar\lambda_{\rm qe}^4 + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1 + \Big[\Big(\bar k^4\bar\lambda_{\rm qe}^4 - 2\bar k^2 2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) \cdot \Big(\bar k^4\bar\lambda_{\rm qe}^4 + 2\bar k^2 2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big]^{1/2}\Big\}^{1/2}, \quad (7)$$

$$\frac{{\rm d}\bar\omega_-}{{\rm d}\bar k} = 2^{-3/2}\Big\{4\bar k^3\bar\lambda_{\rm qe}^4 - \Big[\big(2\bar k^3\bar\lambda_{\rm qe}^4 + 2\bar k\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe})\big)\Big(\bar k^4\bar\lambda_{\rm qe}^4 - 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) + \big(2\bar k^3\bar\lambda_{\rm qe}^4 - 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe})\big) \cdot \Big(\bar k^4\bar\lambda_{\rm qe}^4 + 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big] \Big/ \Big[\Big(\bar k^4\bar\lambda_{\rm qe}^4 - 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) \cdot \Big(\bar k^4\bar\lambda_{\rm qe}^4 + 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big]^{1/2}\Big\} \Big/ \Big\{\bar k^4\bar\lambda_{\rm qe}^4 + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1 - \Big[\Big(\bar k^4\bar\lambda_{\rm qe}^4 - 2\bar k^2 2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big) \cdot \Big(\bar k^4\bar\lambda_{\rm qe}^4 + 2\bar k^2\bar\lambda_{\rm qe}^2(\omega_{\rm pi}/\omega_{\rm pe}) + (\omega_{\rm pi}/\omega_{\rm pe})^2 + 1\Big)\Big]^{1/2}\Big\}^{1/2}, \quad (8)$$

where $\bar\omega_+$ $[\equiv (\omega/\omega_{\rm pe})_+]$ is the scaled higher frequency, $\bar\omega_-$ $[\equiv (\omega/\omega_{\rm pe})_-]$ is the scaled lower frequency, $\bar k$ $(\equiv ka_0)$ is the scaled wave number, $a_0(=\hbar^2/me^2)$ is the Bohr radius of the hydrogen atom, and $\bar\lambda_{\rm qe}$ $(\equiv \lambda_{\rm qe}/a_0)$ is the scaled quantum wavelength.

In order to explicitly investigate the physical properties of both the higher and lower frequency modes of the propagation of the Langmuir oscillations in cold quantum electron-ion plasmas, we set $\omega_{\rm pi}/\omega_{\rm pe} = 1840^{-1/2}$. Figure 1 shows the scaled

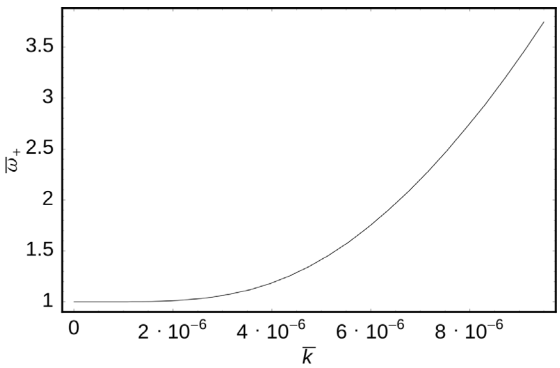

Fig. 1. The scaled frequency $\bar{\omega}_+$ $[\equiv (\omega/\omega_{\rm pe})_+]$ for the higher frequency mode of the propagation of the Langmuir oscillations as a function of the scaled wave number $\bar{k}$ $(\equiv ka_0)$ for $\bar{\lambda}_{\rm qe}$ $(\equiv \lambda_{\rm qe}/a_0) = 2\cdot 10^5$.

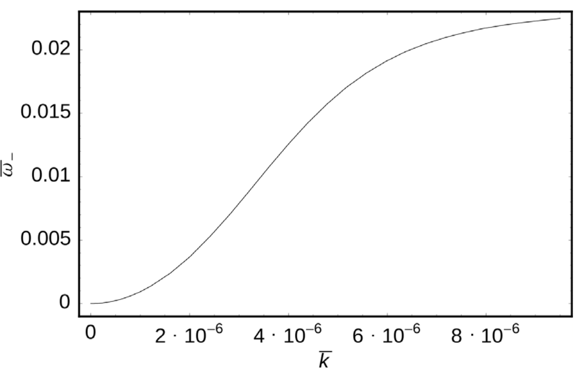

Fig. 2. The scaled frequency $\bar{\omega}_-$ $[\equiv (\omega/\omega_{\rm pe})_-]$ for the lower frequency mode of the propagation of the Langmuir oscillations as a function of the scaled wave number $\bar{k}$ $(\equiv ka_0)$ for $\bar{\lambda}_{\rm qe}$ $(\equiv \lambda_{\rm qe}/a_0) = 2\cdot 10^5$.

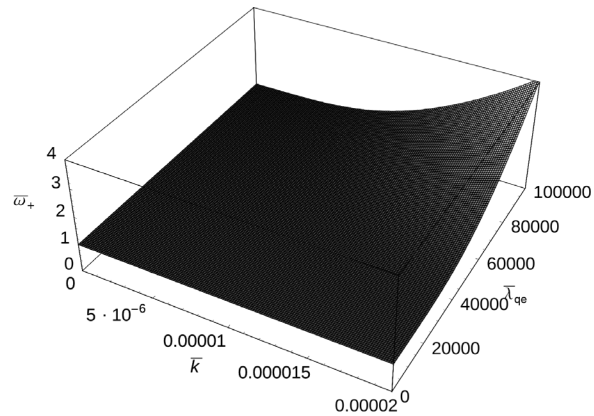

Fig. 3. The surface plot of the scaled frequency $\bar{\omega}_+$ $[\equiv (\omega/\omega_{\rm pe})_+]$ of the higher frequency mode as a function of the scaled wave number $\bar{k}$ $(\equiv ka_0)$ and scaled quantum wavelength $\bar{\lambda}_{\rm qe}$ $(\equiv \lambda_{\rm qe}/a_0)$.

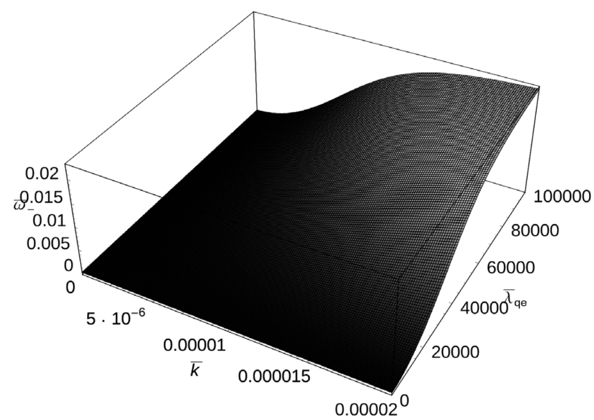

Fig. 4. The surface plot of the scaled frequency $\bar{\omega}_-$ $[\equiv (\omega/\omega_{\rm pe})_-]$ of the lower frequency mode as a function of the scaled wave number $\bar{k}$ $(\equiv ka_0)$ and scaled quantum wavelength $\bar{\lambda}_{\rm qe}$ $(\equiv \lambda_{\rm qe}/a_0)$.

frequency $\bar{\omega}_+$ $[\equiv (\omega/\omega_{\rm pe})_+]$ for the higher frequency mode of the propagation of the Langmuir oscillations as a function of the scaled wave number $\bar{k}$ $(\equiv ka_0)$. Figure 2 represents the scaled frequency $\bar{\omega}_-$ $[\equiv (\omega/\omega_{\rm pe})_-]$ for the lower frequency mode of the propagation of the Langmuir oscillations as a function of the scaled wave number $\bar{k}$. As we see from these figures, the higher frequency mode increases more briskly than the lower frequency mode. It is interesting to note that the higher frequency mode starts from the electron plasma frequency, but the lower frequency mode begins from the zero frequency. Figure 3 represents the surface plot of the higher frequency mode $\bar{\omega}_+$ as a function of the scaled wave number $\bar{k}$ and the scaled quantum wavelength $\bar{\lambda}_{\rm qe}$ $(\equiv \lambda_{\rm qe}/a_0)$. As it is shown, the higher frequency mode increases with an increase of the scaled quantum wavelength. Thus, it is found that the quantum effect enhances the higher frequency mode of the propagation of the Langmuir oscillation. Figure 4 illustrates the surface of the lower frequency mode $\bar{\omega}_-$ as a function of the scaled wave number $\bar{k}$ and the scaled quantum wavelength $\bar{\lambda}_{\rm qe}$. From this figure, it is found that the lower frequency mode is saturated with increasing the scaled wave number. It is also found that the lower frequency mode is saturated with increasing the scaled quantum wavelength. Figure 5 represents the scaled group velocities $\mathrm{d}\bar{\omega}/\mathrm{d}\bar{k}$ of the higher and lower frequency modes as functions of the scaled wave number $\bar{k}$. In this figure, it is shown that the group velocity of the

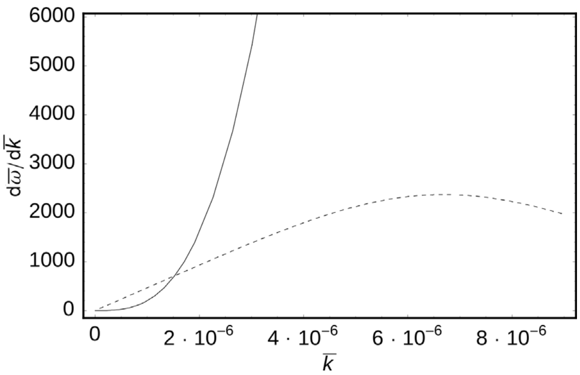

Fig. 5. The scaled group velocities $\mathrm{d}\bar{\omega}/\mathrm{d}\bar{k}$ of the higher and lower frequency modes of the propagation of the Langmuir oscillations as functions of the scaled wave number $\bar{k}$ $(\equiv ka_0)$ for $\bar{\lambda}_{\mathrm{qe}}$ $(\equiv \lambda_{\mathrm{qe}}/a_0) = 10^5$. The solid line represents the scaled group velocity $\mathrm{d}\bar{\omega}_+/\mathrm{d}\bar{k}$ of the higher frequency mode. The dashed line represents the scaled group velocity $\mathrm{d}\bar{\omega}_-/\mathrm{d}\bar{k}$ of the lower frequency mode.

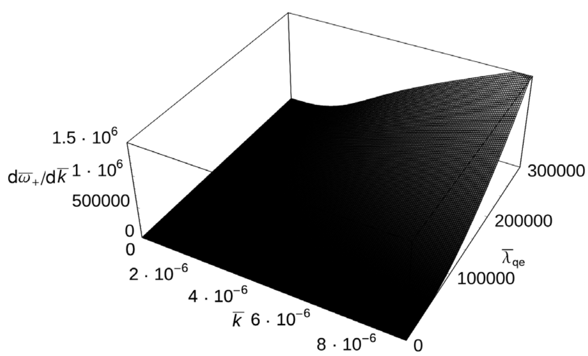

Fig. 6. The surface plot of the scaled group velocity $\mathrm{d}\bar{\omega}_+/\mathrm{d}\bar{k}$ of the higher frequency mode as a function of the scaled wave number $\bar{k}$ $(\equiv ka_0)$ and scaled quantum wavelength $\bar{\lambda}_{\mathrm{qe}}$ $(\equiv \lambda_{\mathrm{qe}}/a_0)$.

higher frequency mode increases with an increase of the scaled wave number. However, it is found that the scaled group velocity of the lower frequency mode has a maximum and decreases with an increase of the wave number. Figure 6 represents the surface plot of the scaled group velocity $\mathrm{d}\bar{\omega}_+/\mathrm{d}\bar{k}$ of the higher frequency mode as a function of the scaled wave number $\bar{k}$ and the scaled quantum wavelength $\bar{\lambda}_{\mathrm{qe}}$. As it is seen, the group velocity of the higher frequency mode increases with an increase of the scaled quantum wavelength. Thus, it is found that the quantum effect

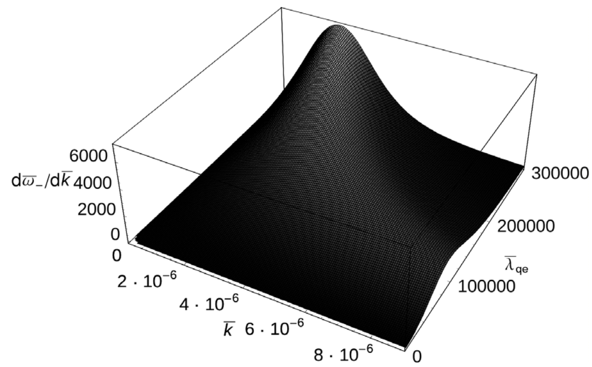

Fig. 7. The surface plot of the scaled group velocity $\mathrm{d}\bar{\omega}_-/\mathrm{d}\bar{k}$ of the lower frequency mode as a function of the scaled wave number $\bar{k}$ $(\equiv ka_0)$ and scaled quantum wavelength $\bar{\lambda}_{\mathrm{qe}}$ $(\equiv \lambda_{\mathrm{qe}}/a_0)$.

enhances the group velocity of the higher frequency mode of the propagation of the Langmuir oscillation. Figure 7 shows the surface plot of the scaled group velocity $\mathrm{d}\bar{\omega}_-/\mathrm{d}\bar{k}$ of the lower frequency mode as a function of the scaled wave number $\bar{k}$ and the scaled quantum wavelength $\bar{\lambda}_{\mathrm{qe}}$. From this figure, we find that the group velocity of the lower frequency mode has a maximum position of the scaled quantum wavelength. In addition, it is interesting to note that the maximum position of the quantum wavelength is shifted to the higher quantum wavelength with decreasing the wave number. Beyond the maximum position of the scaled quantum wavelength, the group velocity is found to be decreased with increasing the quantum effect. Hence, we understand that the contribution of ions plays an important role in wave propagations in cold quantum electron-ion plasmas. These results would be useful for understanding the physical properties of the propagation of Langmuir oscillations in cold quantum electron-ion plasmas.

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